

analytic functions of several complex variables

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When most people think of complex analysis, they picture a single complex variable swirling around the unit circle, the elegance of the Cauchy integral formula, and the beauty of holomorphic functions on the complex plane. Yet the world of complex functions expands dramatically when we allow more than one complex dimension. Analytic functions of several complex variables reveal a landscape rich with surprising phenomena, deep theorems, and powerful techniques that are foundational to modern mathematics. For researchers and students navigating this terrain, the American Mathematical Society's Chelsea Publishing arm has been a pivotal source of authoritative texts and cutting edge research.

In this article we explore the core concepts of several complex variables, highlight landmark results, and showcase how AMS Chelsea Publishing has shaped the field through its curated catalog of books and monographs. Whether you're a graduate student beginning your journey, a seasoned researcher seeking a comprehensive reference, or simply a curious mathematician, this guide offers a thorough, engaging overview that balances theory, history, and practical resources.

The Landscape of Several Complex Variables

Historical Roots

The study of analytic functions in more than one complex variable dates back to the late 19th century, when mathematicians such as **Henri Poincaré** and **Elie Cartan** began to formalize the notion of holomorphicity in higher dimensions. Early pioneers like **Josef Hartogs** discovered that the behavior of functions in multiple variables could differ dramatically from the one variable case, leading to the first instances of what would become central themes in the field.

In the early 20th century, the development of the *Cauchy–Riemann equations* was extended to several variables, giving rise to the concept of **holomorphic functions** on complex manifolds. The 1930s and 1940s saw the emergence of powerful tools such as the **Oka–Weil theorem** and the theory of **domains of holomorphy**, setting the stage for a deeper understanding of complex analytic structures.

Fundamental Concepts

At the heart of several complex variables lies the notion of a function $f: \mathbb{C}^n \rightarrow \mathbb{C}$ that is holomorphic in each variable separately and satisfies the generalized Cauchy–Riemann equations. Unlike the one variable case, the geometry of the domain plays a crucial role: the shape of the domain can affect whether a function can be extended, how singularities behave, and what kinds of boundary values are possible.

Key definitions include:

- Polydiscs – products of discs in each complex coordinate, serving as basic building blocks for local analysis.
- Domains of holomorphy – domains that admit holomorphic functions that cannot be extended beyond them.
- Stein manifolds – complex manifolds that generalize the notion of domains of holomorphy, possessing rich cohomological properties.
- Plurisubharmonic functions – generalizations of subharmonic functions that provide a flexible tool for studying

complex convexity.

Key Theorems and Phenomena

Several complex variables is replete with results that have no one variable analogues:

1. Hartogs' Extension Theorem – In dimensions $(n \geq 2)$, a function holomorphic outside a compact set can often be extended across that set, highlighting the “thickening” effect of higher dimensions.
2. Oka–Weil Approximation Theorem – Provides conditions under which holomorphic functions can be uniformly approximated by polynomials on compact sets.
3. Remmert–Stein Theorem – Describes the structure of analytic sets in complex spaces, revealing the intricate geometry of zeros.
4. Catlin's Theorem on the regularity of the $(\bar{\partial})$ -Neumann problem – Connects complex geometry with partial differential equations, showcasing the interdisciplinary nature of the field.

These results illustrate the rich interplay between analysis, geometry, and topology inherent in the study of analytic functions of several complex variables.

Core Topics in Analytic Functions of Several Complex Variables

Holomorphic Functions and Domains

Holomorphicity in several variables is defined via the vanishing of the complex derivative with respect to each variable. A function (f) is holomorphic on a domain $(D \subset \mathbb{C}^n)$ if, for every point $(z_0 \in D)$, the limit

$$\lim_{h \rightarrow 0} \frac{f(z_0 + h e_j) - f(z_0)}{h}$$

exists for each coordinate direction (e_j) . This local definition leads to powerful global consequences, such as the **Identity Theorem**, which asserts that a holomorphic function that vanishes on a set with an accumulation point must be identically zero.

Power Series and Convergence

Just as in one variable, analytic functions admit power series expansions. For a function (f) holomorphic on a polydisc $(\Delta = \{ |z_j|$

$$f(z) = \sum_{\alpha \in \mathbb{N}^n} a_{\alpha} (z - z_0)^{\alpha}$$

where $(\alpha = (\alpha_1, \dots, \alpha_n))$ is a multi index, and convergence is absolute and uniform on compact subsets. The radius of convergence in each variable is determined by the distance to the nearest singularity, and the interplay between variables can lead to non-trivial convergence domains, such as Hartogs' figure.

Hartogs' Extension Theorem

One of the most striking differences between one variable and several variable complex analysis is the phenomenon captured by Hartogs' theorem. Suppose $(D \subset \mathbb{C}^n)$ with $(n \geq 2)$ is a domain and $(K \subset D)$ is compact. If (f) is holomorphic on $(D \setminus K)$, then (f) extends holomorphically to all of (D) . This result indicates that isolated singularities in higher dimensions are rare and can often be “filled in,” contrasting sharply with the isolated poles that appear in one variable.

Oka–Weil Theorem

The Oka–Weil theorem provides a powerful approximation tool. Let $K \subset \mathbb{C}^n$ be a compact set with polynomially convex hull $\hat{K}$. If f is holomorphic in a neighborhood of K , then for any $\epsilon > 0$ there exists a polynomial P such that

$$\sup_{z \in K} |f(z) - P(z)| < \epsilon$$

Consequently, holomorphic functions on polynomially convex sets can be uniformly approximated by polynomials, a cornerstone of the theory of several complex variables and a key tool in the construction of holomorphic functions with prescribed properties.

Plurisubharmonic Functions

Plurisubharmonic functions generalize subharmonic functions to multiple complex variables. A function $u: D \rightarrow [-\infty, \infty)$ is plurisubharmonic if it is upper semi continuous and its restriction to any complex line is subharmonic. These functions play a central role in complex potential theory, the definition of pseudoconvexity, and the study of the $(\bar{\partial})$ -problem. They also provide a natural setting for defining complex Monge–Ampère equations, which have applications in Kähler geometry and complex dynamics.

AMS Chelsea Publishing: A Legacy in Complex Analysis

About AMS Chelsea Publishing

Founded in 1944, **AMS Chelsea Publishing** is a division of the American Mathematical Society dedicated to publishing high quality monographs, textbooks, and research volumes in mathematics. With a focus on accessibility and depth, the publisher has produced a wealth of titles that serve as foundational texts for both graduate students and researchers. Their catalog spans pure and applied mathematics, with a significant emphasis on complex analysis, functional analysis, and differential geometry.

Notable Publications on Several Complex Variables

AMS Chelsea Publishing has been instrumental in disseminating seminal works on analytic functions of several complex variables. Below are some of the most influential titles that have shaped the field:

- “Several Complex Variables” by W. Rudin – A classic introduction that blends rigorous theory with illustrative examples.
- “Function Theory of Several Complex Variables” by R. Narasimhan – A comprehensive text that covers both foundational concepts and modern developments.

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