

analysis of integrated and cointegrated time series with r

use r

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Introduction

Time series data are everywhere—from daily stock prices and quarterly GDP figures to hourly weather readings and monthly energy consumption. When analysts and researchers want to uncover long term relationships among such data, two concepts become central: **integration** and **cointegration**. Understanding whether a series is integrated, and whether multiple integrated series move together over time, is critical for building reliable econometric models, forecasting, and policy analysis.

This article provides a comprehensive guide to the **analysis of integrated and cointegrated time series with R**. It walks you through the theory, the diagnostic tests, the estimation techniques, and practical R code that makes the entire workflow reproducible. Whether you are a graduate student, a data scientist, or a financial analyst, you will find a step by step approach that can be adapted to a wide range of applications.

What Are Integrated Time Series?

Definition and Properties

An integrated time series is one that becomes stationary after differencing a certain number of times. The order of integration, denoted by d , is the minimum number of differences required to render the series stationary. For example, a series with $d = 1$ (often called $I(1)$) is non stationary in its level form but stationary in its first difference.

Key properties of integrated series include:

- Unit root behavior: The presence of a unit root implies that shocks have permanent effects.
- Mean reversion in differences: While the level may wander, the first difference typically exhibits mean reverting behavior.
- Long memory: Integrated series often display long range dependence, meaning past values influence future observations over extended periods.

How to Test for Integration

Before proceeding with cointegration analysis, you must confirm that each series is integrated at the same order. The most common tests are:

- Augmented Dickey–Fuller (ADF) test: Assesses the null hypothesis of a unit root.
- Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test: Tests the null of stationarity.
- : A non parametric alternative to ADF.

In R, the `urca` and `tseries` packages provide convenient functions to perform these tests.

What Is Cointegration?

Conceptual Overview

Cointegration refers to a statistical relationship where two or more non stationary series share a common stochastic trend. Even though each series individually may wander, their linear combination is stationary. This implies a

long run equilibrium binding them together.

Practical Implications

Cointegration is crucial for:

- Econometric modeling: It justifies the use of Vector Error Correction Models (VECMs).
- Policy analysis: Long term relationships can inform monetary or fiscal policies.
- Financial trading: Pairs trading strategies rely on cointegrated asset pairs.

The Relationship Between Integration and Cointegration

Why Integrated Series Can Be Cointegrated

Two $I(1)$ series can be cointegrated if there exists a linear combination that eliminates the unit root. For instance, the price of a commodity and its exchange rate may drift apart in the short run but revert to a long run equilibrium.

Theoretical Underpinnings

Johansen's maximum likelihood framework generalizes Engle–Granger's two step procedure to multiple series. The key idea is to estimate the rank of the cointegration space, which indicates the number of independent cointegrating relationships.

Tools and Packages in R for Time Series Analysis

- `urca` – unit root and cointegration tests.
- `tseries` – ADF, PP, and KPSS tests.
- `vars` – VAR and VECM estimation.
- `lmtest` – diagnostic tests for regression models.
- `forecast` – forecasting tools for stationary and non stationary series.
- `quantmod` – data acquisition and technical analysis.
- `plm` – panel data extensions of cointegration tests.

Step by Step Guide to Analysis of Integrated and Cointegrated Time Series with R

1. Data Preparation

Load your data, ensure it is time indexed, and convert it into a time series object.

```
library(quantmod)
getSymbols("OIL", src = "yahoo", from = "2000-01-01")
oil_prices <- Cl(OIL)
```

2. Checking for Stationarity

Apply the ADF test to each series.

```
library(urca)
adf_oil <- ur.df(oil_prices, type = "drift", selectlags = "AIC")
summary(adf_oil)
```

3. Determining the Order of Integration

Repeat differencing until the series becomes stationary. Store the differenced series for subsequent tests.

```
oil_diff1 <- diff(oil_prices, differences = 1)
adf_diff1 <- ur.df(oil_diff1, type = "drift", selectlags = "AIC")
summary(adf_diff1)
```

4. Testing for Cointegration

Use the Johansen test for multiple series.

```
library(urca)
data_matrix <- cbind(oil_prices, exchange_rate)
johansen_test <- ca.jo(data_matrix, type = "trace", ecdet = "const", K = 2)
summary(johansen_test)
```

5. Estimating a Cointegrating Relationship

Extract the cointegrating vector and estimate the long run relationship via OLS.

```
beta <- johansen_test@V[,1] # First cointegrating vector
ols_model <- lm(exchange_rate ~ oil_prices, data = data.frame(exchange_rate, oil_prices))
summary(ols_model)
```

6. Building Error Correction Models

Transform the cointegrated system into a VECM.

```
library(vars)
vecm_model <- vec2var(johansen_test, r = 1)
summary(vecm_model)
```

7. Forecasting and Model Diagnostics

Generate forecasts and check residuals for autocorrelation and heteroskedasticity.

```
forecast_vecm <- predict(vecm_model, n.ahead = 12)
plot(forecast_vecm)
bgtest(vecm_model, lag = 12) # Breusch–Godfrey test
```

Practical Example: Analyzing the Relationship Between Oil Prices and Exchange Rates

Oil price movements often influence currency values, especially for oil exporting countries. We demonstrate how to test for cointegration between Brent crude prices and the USD/EUR exchange rate.

```
# Load data
library(quantmod)
getSymbols("BZ=F", src = "yahoo", from = "2005-01-01") # Brent futures
getSymbols("EURUSD=X", src = "yahoo", from = "2005-01-01") # EUR/USD
```

```
brent &lt;- CI(`BZ=F`)
```

```
eurusd &lt;- CI(`EURUSD=X`)
```

```
# Stationarity checks
```

```
adf_brent &lt;- ur.df(brent, type = "drift", selectlags = "AIC")
```

```
adf_eurusd &lt;- ur.df(eurusd, type = "drift", selectlags = "AIC")
```

```
# Johansen test
```

```
data_mat &lt;- cbind(brent, eurusd)
```

```
johansen &lt;- ca.jo(data_mat, type = "trace", ecdet = "const", K = 2)
```

```
summary(johansen)
```

```
# VECM estimation
```

```
vecm &lt;- vec2var(johansen, r = 1)
```

```
summary(vecm)
```

```
# Forecast
```

```
forecast_vecm &lt;- predict(vecm, n.ahead = 6)
```

```
plot(forecast_vecm)
```

The trace test typically rejects the null hypothesis of no cointegration, suggesting a stable long run relationship between oil prices and the EUR/USD exchange rate. The VECM captures both the short run dynamics and the error correction mechanism.

Common Pitfalls and How to Avoid Them

- Ignoring Seasonal Patterns: Seasonal differencing may be necessary before applying unit root tests.
- Overlooking Structural Breaks: Economic crises can alter long run relationships; use tests that allow for breaks.
- Misinterpreting Cointegration Rank: Rely on both trace and maximum likelihood criteria to confirm the number of cointegrating vectors.
- Using Inadequate Lag Lengths: Under or over specifying lags can bias results; use information criteria like AIC or BIC.
- Failing to Check Model Diagnostics: Residual autocorrelation, heteroskedasticity, or non normality can invalidate inference.

Advanced Topics

Nonlinear Cointegration

Real world

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