

algorithms dasgupta chapter 6 solutions

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Introduction

When studying algorithms, one of the most celebrated chapters in the classic text *Algorithms* by Dasgupta, Papadimitriou, and Vazirani is Chapter 6, which focuses on maximum flow and minimum cut problems. This chapter is not only foundational for understanding network flow theory but also serves as a gateway to many advanced topics such as matching, transportation, and scheduling. In this article we present a comprehensive guide to the **algorithms dasgupta chapter 6 solutions** that covers key concepts, step by step problem solutions, common pitfalls, and practical study tips. Whether you are a student preparing for exams, a self learner tackling textbook exercises, or a professional brushing up on flow algorithms, this resource will help you master the material.

Overview of Chapter 6: Maximum Flow and Minimum Cut

Chapter 6 introduces the notion of a flow network—a directed graph where each edge carries a capacity and a source and sink node are specified. The central question is: how much flow can be pushed from the source to the sink without violating capacity constraints? The answer lies in the **maximum flow** value, which, by the Max Flow Min Cut theorem, equals the capacity of the minimum s t cut. The chapter also explores several algorithms for computing maximum flow, ranging from the classical Ford–Fulkerson method to more efficient variants like Edmonds–Karp, Dinic's algorithm, and the preflow push approach.

Key Concepts Covered

- Flow networks, capacities, and feasible flows
- Residual graphs and augmenting paths
- The Max Flow Min Cut theorem
- Ford–Fulkerson and its limitations
- Edmonds–Karp: breadth first search for shortest augmenting paths
- Dinic's algorithm: level graphs and blocking flows
- Preflow push and the highest label strategy
- Capacity scaling and cost scaling for minimum cost flows
- Applications: bipartite matching, network reliability, and transportation

Detailed Solutions to Selected Problems

Below we walk through solutions to some of the most challenging exercises in Chapter 6. Each solution is presented with a clear problem statement, a high level algorithmic strategy, and a line by line explanation of the implementation logic.

Problem 6.1: Basic Flow Computation

****Problem Statement:****

Given a flow network with capacities, compute the maximum flow from source s to sink t using the Ford–Fulkerson method. The network may contain cycles and parallel edges.

****Solution Overview:****

The Ford–Fulkerson algorithm repeatedly searches for an augmenting path in the residual graph and augments

flow along that path by the bottleneck capacity. While the method is conceptually simple, it can suffer from exponential runtime if the capacities are irrational or if a poor path selection strategy is used.

1. Initialize flow $f(e) = 0$ for all edges e .
2. Construct the residual graph G_f based on current flow.
3. While there exists an $s \rightarrow t$ path P in G_f :

Find the minimum residual capacity $c_{\min}$ along P .

4. For each edge e in P , increase forward flow by $c_{\min}$ and decrease backward flow accordingly.
5. Update the residual graph.

Implementation Tips:

Use adjacency lists to store residual capacities; this allows $O(1)$ updates.

Represent reverse edges explicitly to avoid recomputing them during augmentation.

When capacities are integers, the algorithm terminates in at most $O(|E| \cdot C)$ augmentations, where C is the maximum capacity.

Problem 6.2: The Max Flow Min Cut Theorem

Problem Statement:

Prove that the value of a maximum flow equals the capacity of a minimum $s \rightarrow t$ cut in any flow network.

Proof Sketch:

1. **Feasible Flow Implies Upper Bound:**

For any $s \rightarrow t$ cut (S, T) , the net flow out of S cannot exceed the total capacity of edges crossing from S to T . Thus the flow value $\leq$ capacity of the cut.

2. **Construction of Tight Cut:**

After running a maximum flow algorithm, consider the set S of vertices reachable from s in the residual graph. The edges from S to $T = V \setminus S$ have zero residual capacity, meaning they are saturated. The flow across this cut equals the total capacity of those edges, which is exactly the value of the maximum flow.

3. **Conclusion:**

The maximum flow value equals the capacity of this minimum cut, establishing the theorem.

Problem 6.3: Capacity Scaling Algorithm

Problem Statement:

Implement the capacity scaling algorithm to compute maximum flow in $O(|E| \cdot |V| \cdot \log C)$ time, where C is the maximum capacity.

Algorithm Outline:

1. Set scaling parameter $\Delta = \frac{C}{2^k}$
2. While $\Delta \geq 1$

Construct a residual graph considering only edges with residual capacity $\geq \Delta$

Run a blocking flow algorithm (e.g., Dinic) on this restricted graph to find augmenting paths that respect the scaling threshold.

Reduce Δ by half and repeat.

Key Insight:

By focusing on high capacity edges first, the algorithm ensures that each augmentation pushes a large amount of flow, reducing the number of phases needed. The $\log$ factor arises from halving $\sum_{v \in V} c_v$.

Problem 6.4: Bipartite Matching via Flow

Problem Statement:

Given a bipartite graph (U, V, E) , find a maximum cardinality matching using a flow network.

Construction:

Create a super source s and connect it to every vertex in U with capacity 1.

Connect every vertex in V to a super sink t with capacity 1.

For each edge $(u, v) \in E$, add a directed edge $u \rightarrow v$ with capacity 1.

Solution:

Run any maximum flow algorithm (e.g., Edmonds–Karp). Because all capacities are 1, the flow value equals the size of the maximum matching. The set of edges (u, v) with flow 1 corresponds to the matched pairs.

Problem 6.5: Minimum Cost Flow

Problem Statement:

Extend the maximum flow framework to handle costs on edges and find a flow of a specified value with minimal total cost.

Approach:

Use the successive shortest augmenting path (SSAP) algorithm with potentials to avoid negative cycles. The algorithm proceeds as follows:

1. Initialize flow to zero and potentials $\phi(v) = 0$ for all vertices.
2. While the desired flow value k has not been sent:

Run Dijkstra's algorithm on the residual graph with modified edge weights $w'(u, v) = w(u, v) + \phi(u) - \phi(v)$ to find the shortest augmenting path.

3. Augment flow along this path by the minimum residual capacity.
4. Update potentials: $\phi(v) \leftarrow \phi(v) + \text{dist}(v)$ where $\text{dist}(v)$ is the distance from s to v in the modified graph.

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